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Rumor propagation on networks with community structure

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HIGHLIGHTS

- A network model is proposed for generating networks with nonuniform communities.
- Rumor propagation on the generated networks is investigated.
- Bridge hubs have outstanding performance in propagation speed and propagation size.
- Larger modularity can reduce rumor propagation.
- When the decay rate is large, a rumor from a larger community spreads more widely.

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ABSTRACT

In this paper, based on growth and preferential attachment mechanism, we give a network generation model aiming at generating networks with community structure. There are three characteristics for the networks generated by the generation model. The first is that the community sizes can be nonuniform. The second is that there are bridge hubs in each community. The third is that the strength of community structure is adjustable. Next, we investigate rumor propagation behavior on the generated networks by performing Monte Carlo simulations to reveal the influence of bridge hubs, nonuniformity of community sizes and the strength of community structure on the dynamic behavior of the rumor propagation. We find that bridge hubs have outstanding performance in propagation speed and propagation size, and larger modularity can reduce rumor propagation. Furthermore, when the decay rate of rumor spreading β is large, the final density of the stiflers is larger if the rumor originates in larger community. Additionally, when on networks with different strengths of community structure, rumor propagation exhibits greater difference in the density of stiflers and in the peak prevalence if the decay rate β is larger.

1. Introduction

Rumor propagation is an important communication form in our life. Complex network is the main medium for rumor propagation. Traditionally, rumor spreads by word of mouth in social network. In the Internet Age, rumor spreads on SNS (Social Networking Service). The underlying networks on which rumor spreads exhibit a number of characteristics which include the small-world property [1], scale-free degree distributions [2,3], and clustering [1,4,5]. Another crucial characteristic is community structure [4,6,7] and generally, community sizes are nonuniform [8]. It is important to reveal

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how the networks with nonuniform communities affect the propagation dynamics of rumors for the prevention and control of the large-scale spread of the rumor and the maintenance of social stability.

Because the rumor propagation is similar to the epidemic spreading in many ways, the methods and results of epidemic dynamics on complex networks are often used to study rumor propagation on complex networks. Simulation modeling is one way to study the propagation dynamics on complex networks and it contains two aspects: network generation and analysis of propagation dynamics on networks.

To reveal the effect of community structure on propagation dynamics, lots of network models which can generate community structure have been proposed [9–19] and dynamics of epidemic spreading on these networks have been studied [11–18]. In [9], starting from an initial network, at each time step, a node is added and enters to a community randomly. The newly added node connects with nodes in the same community based on inner-degree and connects with nodes in the other communities based on inter-degree. The analysis results show the networks generated are scale free networks with communities whose sizes are nearly uniform. The networks generated by the network model in [9] are called SFcN (Scale-free Networks with Communities) in [14]. By the switching algorithm, Huang et al. obtained the networks called SFN (Scale-free Networks without Communities) having the same degree distribution to SFcN's but without community structure. On the networks SFcN and SFN, Huang et al. [14] studied epidemic spreading based on the Susceptible-Infected (SI) epidemic model. Comparing the results in these two kinds of networks, it is found that strong community structure can reduce epidemic danger. The network generation model in [10] is different from the one in [9]. The newly added node connects with nodes in the same community and nodes in different communities both by preferential attachment mechanism based on total-degree. The weights of edges are express by nodes' degrees, and the internal edges and external edges are weighed by two exponents. It is shown that the networks generated by network model in [10] are weighted scale free networks with communities whose sizes are nearly uniform. Based on the SI epidemic model, Chu et al. [10] found that the external weighting exponent plays a much more important role in slackening the epidemic spreading and reducing the danger brought by the epidemic than the internal weighting exponent. Moreover, they found that the strong community structure is no longer helpful for slackening the danger brought by the epidemic in the weighted cases. In [11], each community has the same number of nodes. Intra-community links are generated at higher probability than that for generating inter-community links. Liu et al. [11] discussed the epidemic spreading on such random networks based on Susceptible-Infected-Susceptible (SIS) model. They found that the epidemic threshold will increase with the decrease of the degree of community. Wu et al. [12] gave a model with adjustable clustering coefficient and degree of community, and they discovered the final size of infected nodes depends mainly on the degree of community. Huang et al. [13] investigated the dynamics of information propagation on modular networks. They found the life span of information can be maximized by the number of modules.

Of course, there are also differences between epidemic models and rumor models—most obviously in the mechanism of removal, i.e., the event that a spreader ceases to spread the rumor. In the standard models of rumor spread called Daley—Kendal (DK) model [20] and Maki—Thompson (MT) model [21], the mechanism of removal is that the spreaders are removed with some probability if they contact other spreaders or stiflers. While in epidemic models, the spreaders are removed with some probability whether they contact other spreaders or stiflers or not. Recently, many researches investigate rumor propagation on complex networks based on MT rumor model to reveal the influence of complex network topology on dynamics of rumor propagation. Zanette investigated the rumor propagation on both static [22] and dynamic [23] small-world networks by numerical analysis. His studies showed that at a finite randomness of the underlying small-world network, the rumor propagation exhibits a critical transition between a regime where the rumor dies in a small neighborhood of its origin, and a regime where it spreads over a finite fraction of the whole population. Moreno et al. studied the rumor propagation on SF networks [24,25]. Their studies revealed the network topology interact with the rules of the rumor model and highlighted the great impact of network heterogeneity on the dynamics of rumor propagation. There are also many researches analyzing the mechanisms of rumor propagation. Nekovee et al. [26] considered a forgetting mechanism which leads to presence of propagation threshold on Small world network and finite SF networks. Han et al. [27] proposed energy model to study the dynamics of rumor spreading on synthetic networks and real-world social networks.

The researches [9–19] pay close attention to the influence of networks with uniform communities on dynamics of epidemic spreading. The influence of nonuniformity of community sizes on propagation dynamics is neglected. For dynamics of rumor propagation, there is little work taking into account the impact of community structure on rumor propagation.

Considering these scenarios, in this paper, we investigate rumor propagation on networks with nonuniform communities. Firstly, we propose a network generation model which can describe social networks realistically. Next, we explore how the network characteristics, such as bridge hubs and the community structure, influence the dynamic behavior of rumor spreading. Additionally, we investigate how the influence will be affected by the value of the propagation parameters.

The rest of this paper is organized as follows. In Section 2, we propose a network evolving model. We analyze the properties of the networks in Section 3. Based on rumor propagation model, we perform Monte Carlo simulations on networks with community structure and analyze the effect of community structure on rumor propagation in Section 4. Finally, we conclude the paper in Section 5.

2. Network generation model

In this section, based on growth and preferential attachment mechanism, we propose a network evolving model aiming at generating networks with nonuniform communities. The generation algorithm for the model is described below:

(i) Initialization

The network is initialized with M communities denoted by $C_1, C_2, \ldots, C_M$. In each community, there are m_0 fully connected nodes. Additionally, for any two communities, there is an inter-community link between two nodes selected from the two communities respectively.

(ii) Growth

At each time step, a new node is added into the network. The newly added node enters community C_j with probability p_j and $\sum_j p_j = 1$. The newly added node will connect to m $(1 \leq m \leq m_0)$ nodes in community C_j according to the preferential attachment mechanism, which means the probability $\prod_j(l)$ that the newly added node will connect to node l $(l \in C_j)$ depends on the degree k_{lj} of node l in the community C_j. That is $\prod_j(l) = \frac{k_{lj}}{\sum_r k_{rj}}$. At the same time, n $(1 \leq n \leq m)$ operations will be executed with probability α so that the newly added node will connect to the other $M - 1$ communities through inter-community links. In each operation, first, community C_h is chosen with probability p_h, and then the newly added node will connect to a node in community C_h following the preferential attachment mechanism referred above for $h \neq j$ and will not connect to nodes in community C_h for $h = j$.

In our model, growth and preferential attachment are two main evolution mechanisms which lead to a scale-free feature of networks [2]. In addition, we give the probability distribution $p_1, p_2, \ldots, p_M$ and a new node is added to the community C_j with probability p_j $(j = 1, 2, \ldots, M)$. In this way, the community-size distribution of the generated network can approximate the expected distribution $p_1, p_2, \ldots, p_M$ which can be either nonuniform or uniform. Finally, the given probability distribution $p_1, p_2, \ldots, p_M$ plays another role. When a newly added node in community C_j emits inter-community links, whether it will connect to the community C_h is also determined by the probability p_h $(h \neq j)$ corresponding to the community C_h $(h \neq j)$. So, for links emitting from a node in community C_j, the probability p_h $(h \neq j)$ can be regarded as the rate of attraction to community $C_h (h \neq j)$. This is the case in reality that the larger community can attract more inter-community links. Thus, based on our model, the generated networks can display some generic features of a number of real-world networks. In existing models in Refs. [9–19], however, a new node is added to a randomly selected community. By that way, the community-size distribution of the generated network can only approximate a uniform distribution. So our model can describe social networks realistically.

In the following, we will perform numerical simulations to show that we can generate networks with community structure by the proposed model.

Example. Consider the case of $M = m_0 = m = 3$, $p_1 = \frac{1}{6}$, $p_2 = \frac{1}{3}$, $p_3 = \frac{1}{2}$, $\alpha = 0.1$, $n = 1$ and a total of $N = 500$ nodes. Show that the generated network exhibits community structure.

According to the given conditions, the initial network has three communities denoted by C_1, C_2, C_3. In each initial community, there are three nodes which are fully connected and for each two communities, there is an inter-community link to connect them. Starting from the initial network, at each time step, a new node with 3 intra-community links is added into a community selected by the above probability distribution. In detail, for example, a new node enters community C_1 with probability $p_1 = \frac{1}{6}$. Then, it connects to 3 nodes in community C_1. At the same time, if the random number is smaller than $\alpha = 0.1$, then we execute 1 operation. That is the newly added node connects to a node in community C_2 with the probability $p_2 = \frac{1}{3}$ and connects to a node in community C_3 with the probability $p_3 = \frac{1}{2}$ and it will not connect with the probability $p_1 = \frac{1}{6}$. We repeat the process until the size of the network is up to 500.

As showed in Fig. 1, the generated network exhibits community structure.

In the next section, we will study other important properties of the generated networks.

3. Topological properties of the generated networks

In this section, we mainly investigate three important properties: degree distribution, network modularity [28], and average shortest path length. Firstly, we use the mean-field method [29] to analyze the degree distributions including the degree distribution that is defined in [9] for any community and the degree distribution of the generated networks. Secondly, we analyze how the strength of community structure of the generated network varies with the network configuration parameter α. Finally, we discuss the influence of the network configuration parameter α on the average shortest path length.

3.1. Degree distribution

Let $k_{ij}(t)$ be the degree of the node i in community C_j at tth time step. Let $P_j(k, t)$ be the degree distribution of the community C_j at tth time step.

Theorem 3.1. *For each community C_j $(j = 1, 2, \ldots, M)$, its steady-state degree distribution exists, and is given by*

$$P_j(k) = \lim_{t \to \infty} P_j(k, t) = 2[m + (1 - p_j)\alpha n]^2 k^{-3} \ (k \geq m).$$

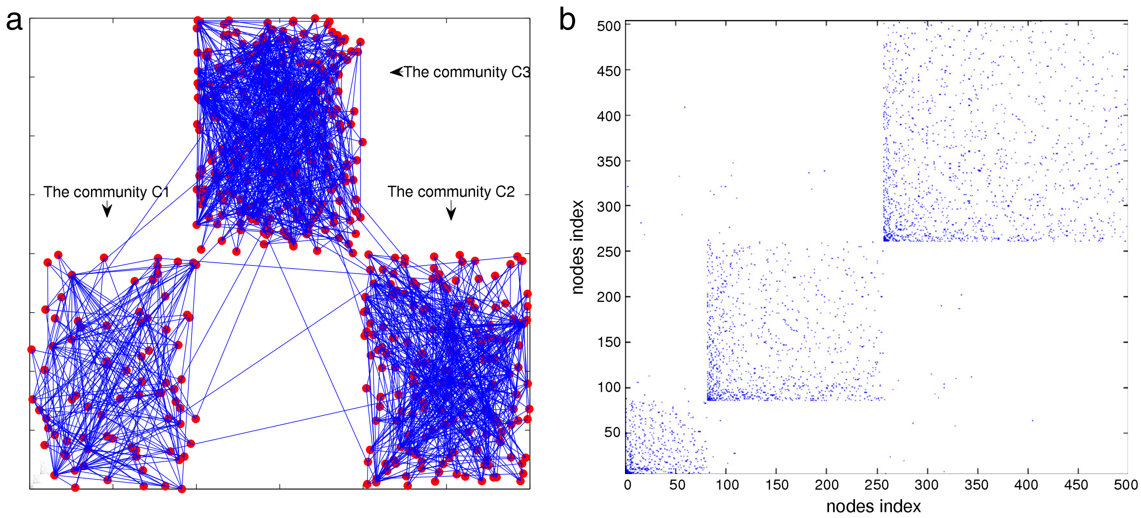

Fig. 1. A network generated by the proposed model: $N = 500, M = 3, m_0 = 3, m = 3, p_1 = \frac{1}{6}, p_2 = \frac{1}{3}, p_3 = \frac{1}{2}, n = 1, \alpha = 0.1$. (a) Graph representation for the generated network. (b) Sparsity pattern of the adjacent matrix for the generated network.

Proof. Assume $k_{ij}(t)$ is continuous, then the rate of the change of $k_{ij}(t)$ can be written as

$$\frac{\partial k_{ij}(t)}{\partial t} = p_j m \frac{k_{ij}}{\sum_l k_{lj}} + \alpha n \sum_{h \neq j} p_h p_j \frac{k_{ij}}{\sum_l k_{lj}}. \tag{1}$$

Since $\sum_l k_{lj} = m_0(m_0 - 1) + 2p_j mt + 2p_j(1 - p_j)\alpha nt$, when t is enough large, $\sum_l k_{lj} \approx 2p_j mt + 2p_j(1 - p_j)\alpha nt$. Thus we have

$$\frac{\partial k_{ij}(t)}{\partial t} = \frac{k_{ij}}{2t}. \tag{2}$$

Note that the node i is added to the community C_j at time t_{ij} with degree $m + (1 - p_j)\alpha n$, so the initial condition of the Eq. (2) is $k_{ij}(t_{ij}/p_j) = m + (1 - p_j)\alpha n$. Solving (2) with its initial condition, we obtain that

$$k_{ij}(t) = [m + (1 - p_j)\alpha n] \left(\frac{p_j t}{t_{ij}}\right)^{\frac{1}{2}}. \tag{3}$$

From Eq. (3), the probability that the node i in community C_j has degree k_{ij} smaller than k is

$$P_j(k_{ij}(t) < k) = P_j\left(t_{ij} > \frac{[m + (1 - p_j)\alpha n]^2 p_j t}{k^2}\right). \tag{4}$$

We assume that we add the nodes at equal time intervals to the community C_j, then the probability density of t_{ij} is given by

$$P_j(t_{ij}) = \frac{1}{m_0 + p_j t}. \tag{5}$$

From (4) and (5), we have

$$P_j\left(t_{ij} > \frac{[m + (1 - p_j)\alpha n]^2 p_j t}{k^2}\right) = 1 - P_j\left(t_{ij} \leq \frac{[m + (1 - p_j)\alpha n]^2 p_j t}{k^2}\right) = 1 - \frac{[m + (1 - p_j)\alpha n]^2 p_j t}{k^2(m_0 + p_j t)}. \tag{6}$$

Computing the derivatives of the functions on both sides of Eq. (6) with respect to k, we can obtain the probability density

$$P_j(k, t) = \frac{2[m + (1 - p_j)\alpha n]^2 p_j t}{m_0 + p_j t} k^{-3}. \tag{7}$$

Taking limits on both sides of Eq. (7), it is easy to get $P_j(k) = \lim_{t \to \infty} P_j(k, t) = 2[m + (1 - p_j)\alpha n]^2 k^{-3}$. The proof is completed.

From (7), we can see that the degree distribution of each community obeys a power law distribution $P_j \sim k^{-\gamma}$, where $\gamma = 3$ independent of M and m.

It is difficult to analyze the degree distribution of the global network by the mean-field method. We predict that it is a power-law distribution $P(k) \sim k^{-3}$ and verify it by numerical simulation.

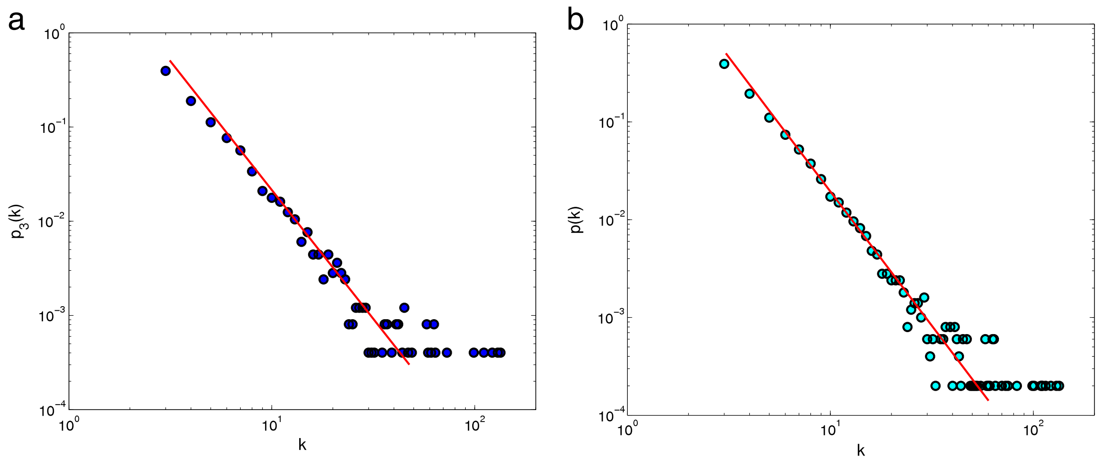

Fig. 2. (a) Log–log representation of the degree distribution of the community C_3. The straight line is plotted according to Eq. (7) but power exponent of "k" is −2.9. (b) Log–log representation of the degree distribution of the network. The slope of the straight line is −2.9.

In the following, we perform numerical simulations for the degree distributions. For a network with $M = m_0 = m = 3$, $p_1 = \frac{1}{6}, p_2 = \frac{1}{3}, p_3 = \frac{1}{2}, n = 1, \alpha = 0.1$ and $N = 5000$, we calculate the distributions of the community C_3 and the degree distribution of the total network, then display them in Fig. 2.

From Fig. 2, we can see there is small deviation between numerical simulations and analytical solutions. It is due to the mathematical approximation of the boundary conditions and the effect of the finite network size in the simulations [28].

3.2. Modularity measure

In [28], Newman et al. have proposed modularity to measure the strength of community structure for complex networks. It is defined as

$$Q = \sum_{i=1}^{M}(e_{ii} - a_i^2),$$

where e_{ij} is the fraction of all edges in the network that link nodes in community i to nodes in community j and $a_i = \sum_j e_{ij}$. According to our network generation model, we have

$$e_{ii} = \frac{p_i m}{m + (1 - \sum_{i=1}^{M} p_i^2)\alpha n},$$

and

$$a_i = \frac{p_i[m + (1 - p_i)\alpha n]}{m + (1 - \sum_{i=1}^{M} p_i^2)\alpha n}.$$

So we can obtain

$$Q = \frac{m}{m + (1 - \sum_{i=1}^{M} p_i^2)\alpha n} - \sum_{i=1}^{M} \frac{p_i^2[m + (1 - p_i)\alpha n]^2}{[m + (1 - \sum_{i=1}^{M} p_i^2)\alpha n]^2}. \tag{8}$$

Then we can obtain the following theorem:

Theorem 3.2. *The modularity Q of the network generated by the generation model is a decreasing function of variable α.*

Proof. Computing the derivatives of the functions on both sides of Eq. (8) with respect to α, we have

$$\begin{aligned}\frac{\partial Q}{\partial \alpha} = &-\frac{mn}{[m + (1 - \sum_{i=1}^{M} p_i^2)\alpha n]^3}\Bigg[\left(m + \left(1 - \sum_{i=1}^{M} p_i^2\right)\alpha n\right)\left(1 - \sum_{i=1}^{M} p_i^2\right) \\ &+ 2(m + \alpha n)\left(\sum_{i=1}^{M} p_i^2 \sum_{i=1}^{M} p_i^2 - \sum_{i=1}^{M} p_i^3\right) - 2\left(\sum_{i=1}^{M} p_i^2 \sum_{i=1}^{M} p_i^3 - \sum_{i=1}^{M} p_i^4\right)\alpha n\Bigg].\end{aligned}$$

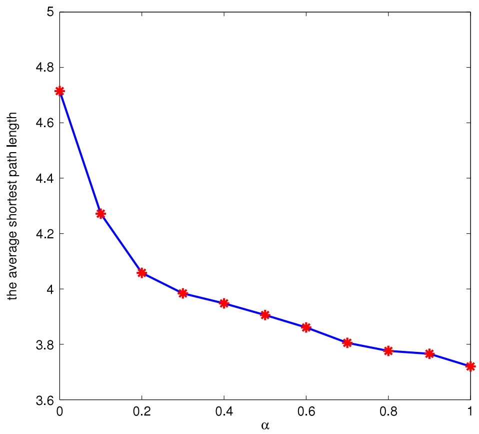

Fig. 3. Average shortest path length versus network configuration parameter α.

For the fixed m and n, and any given α, the partial derivative function $\frac{\partial Q}{\partial \alpha}$ is continuous on the domain $\{(p_1, p_2, \ldots, p_M)| \sum_{i=1}^{M} p_i = 1, p_i \geq 0 (i = 1, 2, \ldots, M)\}$. So $\frac{\partial Q}{\partial \alpha}$ has maximum and minimum on the domain. Applying the Lagrange multiplier method, $\frac{\partial Q}{\partial \alpha}$ can take minimum when $p_1 = p_2 = \cdots = p_M = \frac{1}{M}$ and take maximum when some $p_i (i = 1, 2, \ldots, M)$ is 1 and the others are 0. Thus, for any $\alpha \in [0, 1]$, $\frac{\partial Q}{\partial \alpha} \leq 0$. That is the modularity Q is a decreasing function of variable α. The proof is completed.

3.3. Average shortest path length

Average shortest path length is an important characteristic. Small average shortest path length shows small-world property of the network [3]. In the following, we will show the dependence of average shortest path length on the variable α (from 0 to 1).

Setting $N = 3000$, $M = 3$, $m = 3$, $p_1 = \frac{1}{6}$, $p_2 = \frac{1}{3}$ and $p_3 = \frac{1}{2}$, we generate 50 networks whose sizes are 3000 and take the average of the average shortest path lengths of the 50 generated networks. Fig. 3 shows that the average shortest path length decreases with increasing α. And the average shortest path length is smaller than 4.8. This indicates that the networks show small-world property.

Overall, because of preferential attachment mechanism, in each community, there is a node that achieves more intra-community links and inter-community links than the others. It plays a dual role, community hub [30] and community bridge [31,32]. So it is called bridge hub [32]. It is important for epidemic spreading between communities [32,33]. In rumor propagation, we will pay additional attention to bridge hubs. We can generate networks with nonuniform communities by setting the probability distribution $p_1, p_2, \ldots, p_M$. Additionally, we can adjust the strength of community structure by vary the probability α.

In the following section, we will reveal the influence of bridge hubs, nonuniformity of community sizes and the strength of community structure on dynamic behavior of rumor propagation.

4. Simulations and results of rumor propagation on the generated networks

In this section, we perform extensive Monte Carlo simulations to reveal how the network topology influences the dynamic behavior of rumor propagation. In detail, we set network configuration parameters $p_1 = \frac{1}{6}$, $p_2 = \frac{1}{3}$, $p_3 = \frac{1}{2}$, $m = 3$, $n = 1$ and $M = 3$, and vary values of network configuration parameter α to generate networks, then let a rumor spread on these networks. By comparing the results, we reveal the influence of bridge hubs and the strength of community structure on dynamic behavior of rumor propagation. Additionally, let network configuration parameters $p_1 = \frac{1}{10}$, $p_2 = \frac{1}{10}$, $p_3 = \frac{8}{10}$, $m = 3$, $n = 1$, $M = 3$ and $\alpha = 0.2$. We generate networks whose communities' size is very nonuniform. On these networks, when a rumor originates from different communities, we observe the influence of nonuniformity of community sizes on rumor spreading.

The rumor model is defined as in [25]. Each node in networks plays one of three roles: ignorant, spreader, and stifler. The ignorant is one who has not heard the rumor. The spreader is active to spread the rumor and the stifler is an individual who knows the rumor but loses interest in spreading it. At the beginning, only one node is a spreader and all the other are ignorant. At each time step, each spreader contacts his(her) neighbors. When an ignorant meets a spreader, the ignorant turns into a spreader with probability λ; when a spreader encounters another spreader or a stifler, the spreader loses interest in spreading the rumor and becomes a stifler with probability β. Until there is no spreader, the propagation progress terminates.

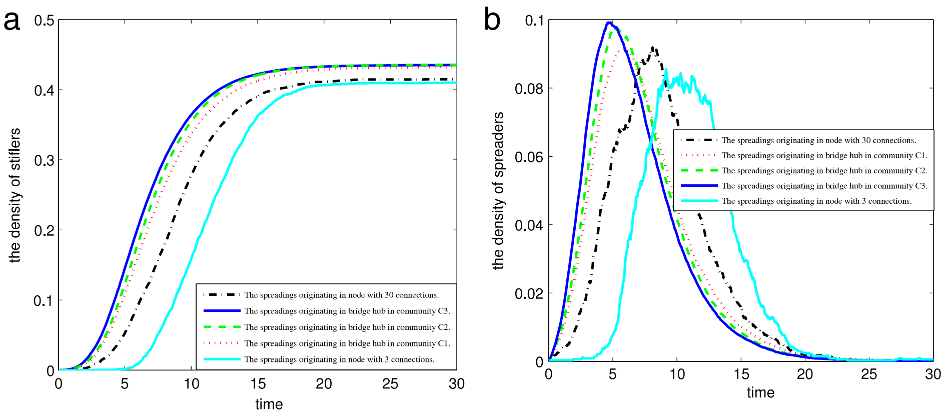

Fig. 4. The time evolutions of (a) the average density of stiflers and (b) the average density of spreaders for rumor propagation with spreading rate $\lambda = 0.2$ and the decay rate $\beta = 0.2$.

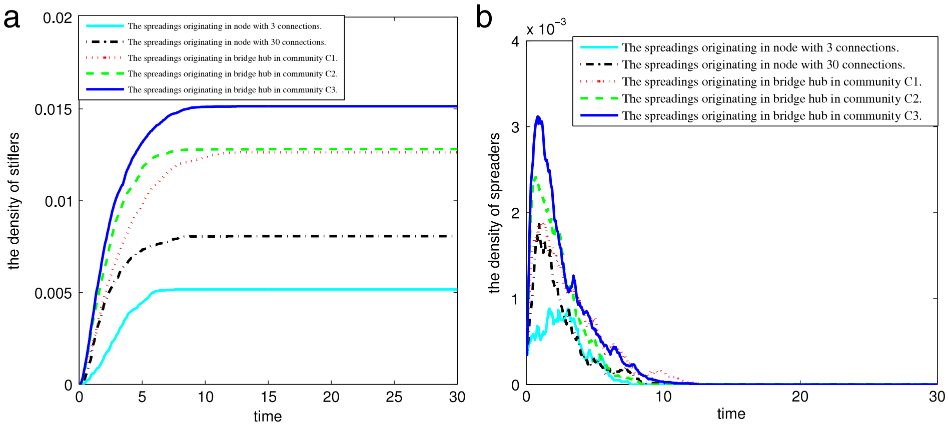

Fig. 5. The time evolutions of (a) the average density of stiflers and (b) the average density of spreaders for rumor propagation with spreading rate $\lambda = 0.2$ and the decay rate $\beta = 1$.

In a rumor spreading process, we mainly pay attention to three quantities: the final density of stiflers (the final rumor size), the maximum density of spreaders at some point in the process (the peak prevalence), and the number of time steps that rumor propagation takes from the first infected case to the peak prevalence case. In the following, we reveal the effect of network topology on rumor propagation by these three quantities.

To quantify the importance of bridge hubs on the dynamics of rumor spreading when the spreading originates in a single node, we compare spreading ability of the bridge hub located in each community, nodes with 30 connections and nodes with minimum degree 3 in networks. Firstly, let $p_1 = \frac{1}{6}, p_2 = \frac{1}{3}, p_3 = \frac{1}{2}, m = 3, n = 1, M = 3$ and $\alpha = 0.2$, we generate 10 scale-free networks whose size are all 3000. In each network, there are three communities C_1, C_2, and C_3. In each community, there is a node with maximum degree and it is a bridge hub. We denote them by H_1, H_2 and H_3 respectively. Next, for each network, we choose H_1, H_2, H_3, a node with 30 connections, and a node with 3 connections to be infected seed nodes individually. For each infected seed node, simulations are implemented for 100 times. So there are 1000 realizations for each of H_1, H_2, H_3, node with 30 connections and node with 3 connections. We average over the results of the 1000 realizations respectively to be spreading ability of H_1, H_2, H_3, node with 30 connections and node with 3 connections. From Figs. 4 and 5, we can find that if the infected seed node is a bridge hub, the rumor size is higher compared with the case when the infected seed is a node with 30 connections or a node with 3 connections. We also find that the rumor size is higher when the degree of bridge hub is larger. The same is true for the peak prevalence. Furthermore, if the infected seed node is a bridge hub, the peak prevalence is reached faster. The phenomenon coincides with what was observed in epidemic models like the SIR model in [34,35], where the final prevalence depends on the degree of the initially infected individuals, but opposes what was observed in rumor propagation in [25,36], where the final influence of rumor reaches the same level irrespective of the degree of the first spreader.

However, in networks with uniform communities, if the rumor originates from the hub with largest degree in different communities, there is little difference for both propagation size and propagation speed. It is shown in Figs. 6 and 7.

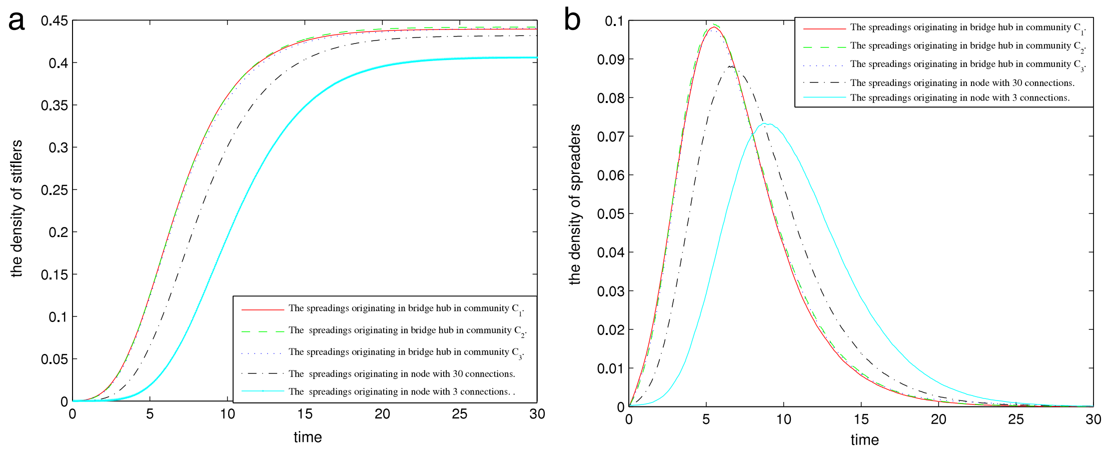

Fig. 6. The time evolutions of (a) the average density of stiflers and (b) the average density of spreaders for rumor propagation with spreading rate $\lambda = 0.2$ and the decay rate $\beta = 0.2$ on generated networks with $p_1 = p_2 = p_3 = 1/3$.

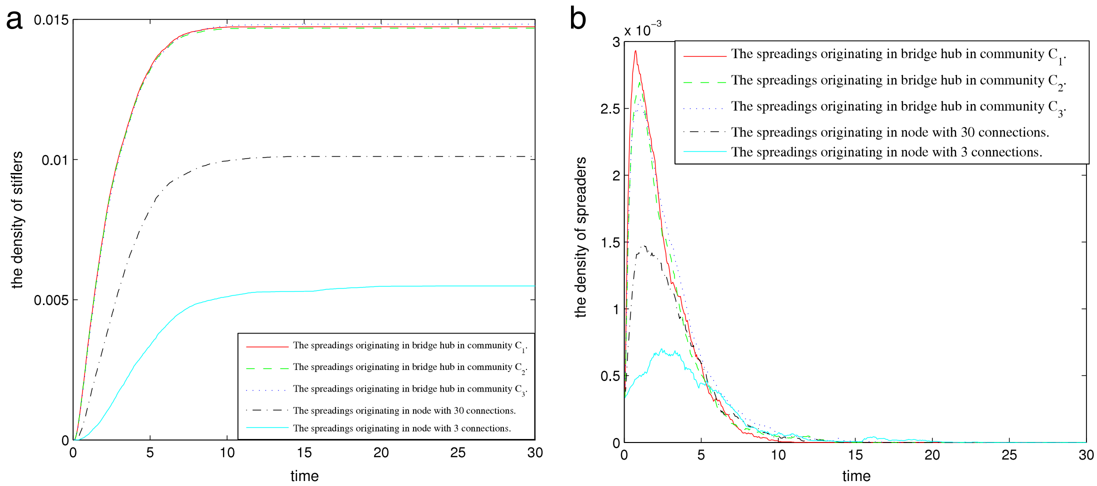

Fig. 7. The time evolutions of (a) the average density of stiflers and (b) the average density of spreaders for rumor propagation with spreading rate $\lambda = 0.2$ and the decay rate $\beta = 1$ on generated networks with $p_1 = p_2 = p_3 = 1/3$.

Generally, the community sizes are nonuniform [37]. Some communities are large, but some are small. When a rumor originates from different communities, whether the final density of stiflers is same is a neglected question. In the following, we will reveal the influence of nonuniform communities on rumor propagation.

Let network configuration parameters $p_1 = \frac{1}{10}$, $p_2 = \frac{1}{10}$, $p_3 = \frac{8}{10}$, $m = 3$, $n = 1$, $M = 3$ and $\alpha = 0.2$, we generate 10 networks where the sizes of the largest community and the smallest one differ nearly by an order of magnitude. On each network, from each community, we select 5% nodes and let each of these nodes to be infected seed to spread a rumor 100 times. We average results of rumor spreading originating in each community on the 10 networks and display in Fig. 8. It can be seen that when the decay rate is small, there is no difference in the final densities, but when the decay rate is large, the final density of stiflers is larger if the rumor originates in larger community.

To inspect the role of the strength of community structure on rumor spreading, we set parameters of network $p_1 = \frac{1}{6}$, $p_2 = \frac{1}{3}$, $p_3 = \frac{1}{2}$, $M = m_0 = m = 3$ and $\lambda = 0.2$. Let the network configuration parameter α and the decay rate for rumor spreading β vary. Due to computation cost, simulations were implemented averaging over 10 different network configurations, and for each configuration we took 100 different realizations with exactly one randomly chosen node infected. Figs. 9–11 all show the time evolutions of average densities of stiflers and spreaders on networks with different network configuration parameter α and same β. Form Figs. 9 to 11, each figure indicates that larger α leads to higher rumor size, larger peak prevalence and shorter time it take for rumor spreading from beginning to peak prevalence needs. Making a comprehensive survey, the influence of α on rumor spreading is affected by the decay rate β. With fixed λ, increasing β will exaggerate the influence of α on rumor spreading. It is an interesting and reasonable phenomenon. Note that the process of rumor spreading is composed of spreader generation and spreader decay. When β is larger, i.e. the spreaders decay faster,

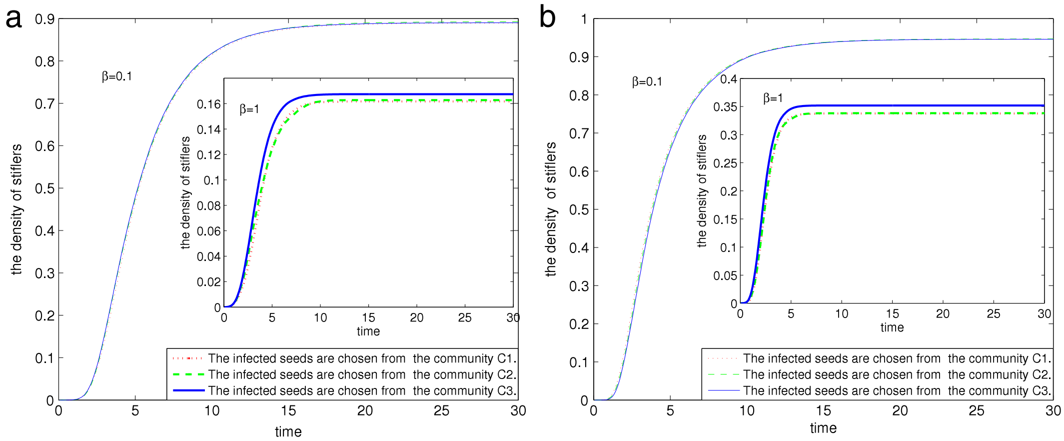

Fig. 8. The average density of stiflers. (a) The spreading rate $\lambda = 0.5$. The decay rate $\beta = 0.1$ (main figure) and the decay rate $\beta = 1$ (inset). (b) The spreading rate $\lambda = 0.8$. The decay rate $\beta = 0.1$ (main figure) and the decay rate $\beta = 1$ (inset).

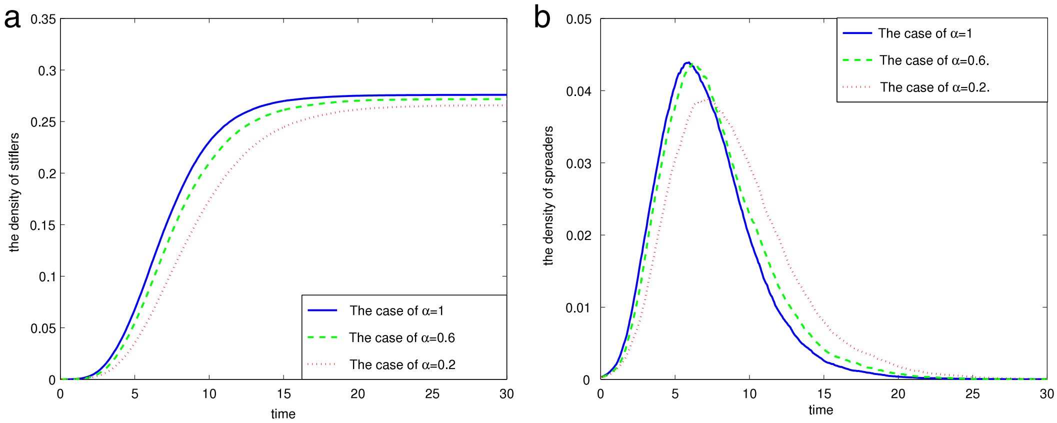

Fig. 9. The time evolutions of average densities of stiflers and spreaders on networks with different network configuration parameter α. The spreading rate is $\lambda = 0.2$ and the decay rate is $\beta = 0.3$.

then the spreaders have few time to spread the rumor. It indicates they can only spread the rumor to relative near nodes. Note that, increasing α can lead to decreasing of average length of shortest path lengths. So in this case, increasing α can promote rumor spreading.

5. Conclusions

Community structure and the heterogeneity of community size are prevalent phenomena in the real-world social networks, such as the friendship network, collaboration network of scientists [4,6–8]. Many researches have shown that networks with uniform communities have large influence on dynamic behavior of epidemic spreading. However, the issue whether networks with nonuniform communities have the same influence on dynamic behavior is neglected.

In this paper, to provide insights to the question, we first give a network evolving model. The networks generated by the model exhibit community structure and the community sizes can be nonuniform. In each community, there is a node which is not only a hub node but also a bridge node. It is so called bridge hub. And the strength of community structure is decreasing when increasing only the values of network configuration parameter α. Additionally, the proposed model may be applied in reality. By sampling from a real-world network with an appropriate scale, we can get some characteristics of the network, such as the empirical distribution of community-size, network configuration parameter α, and so on. By using these parameters in the proposed model, we may produce a large-scale simulated network resembling the real-world network. On the generated networks, we can study the dynamic behavior to provide guidance for designing strategy to control disease and rumor spreading or enhance information spreading.

Based on the network model, we perform Monte Carlo Simulations to display dynamic behavior of rumor spreading on the generated networks. Our main purpose is to reveal how the network characteristics, the bridge hubs, nonuniformity of

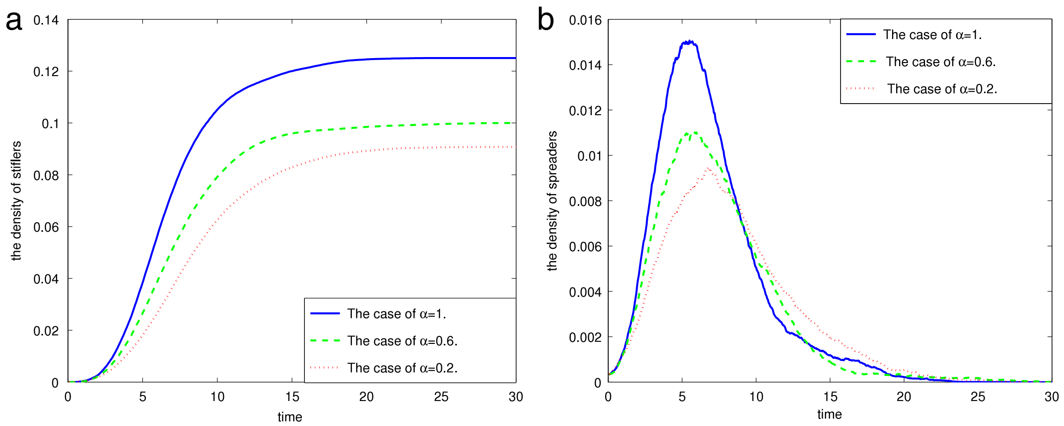

Fig. 10. The time evolutions of average densities of stiflers and spreaders on networks with different network configuration parameter α. The spreading rate is $\lambda = 0.2$ and the decay rate is $\beta = 0.5$.

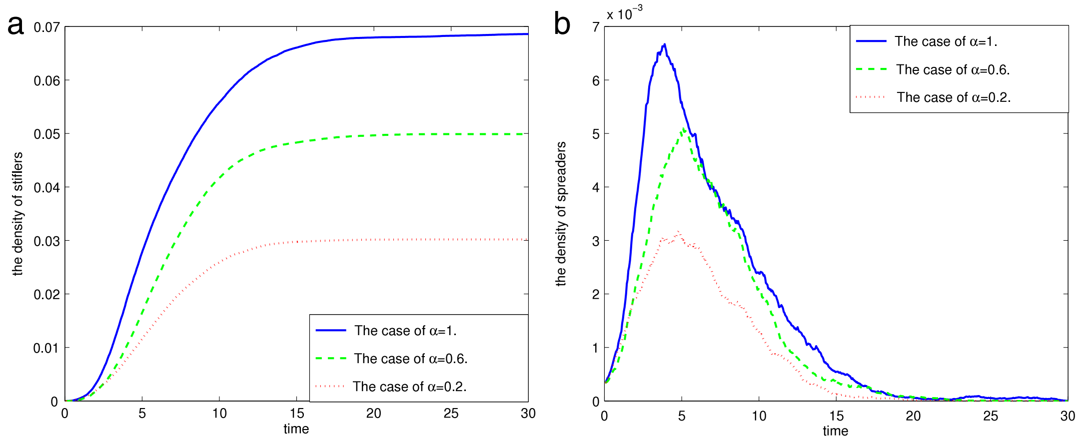

Fig. 11. The time evolutions of average densities of stiflers and spreaders on networks with different network configuration parameter α. The spreading rate is $\lambda = 0.2$ and the decay rate is $\beta = 0.7$.

communities and the strength of community structure, influence the dynamic behavior of rumor propagation. Additionally, we reveal how the influence is affected by the propagation mechanism, in fact, the propagation parameters. From the simulations, we have the following conclusions:

(1) As an infected seed, a bridge hub has outstanding performance in propagation speed and propagation size.
(2) When the community sizes are nonuniform obviously, then if the decay rate of rumor spreading is large, the rumor originates from the larger community, the final density of stiflers is larger.
(3) When the strength of community structure decreases, the rumor size and the peak prevalence increase, and the time which the rumor propagation process from the first infected case to the peak prevalence case take decreases.
(4) when on networks with different strengths of community structure, rumor propagation exhibits greater difference in the density of stiflers and the peak prevalence when the decay rate of rumor propagation is large.

From the aforementioned conclusions, we can gain an insight into rumor spreading on networks with community structure and we can choose effective control strategies and targeted immunization methods (see [33,38] and references therein). Further, the results inspire us to understand other social phenomena such as the spreading of new ideas or the design of efficient marketing campaigns.

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